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# THE MATHEMATICS TEACHER

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J. D'ALEMBERT

Secrétaire perpétuel de l'Académie Française, membre de l'Académie des Sciences &c. &c. &c.

*Dédié à Monsieur de Voltaire.*

*Ce Sage à l'humanité rend un culte secret,  
Se dérobe à la gloire et se cache à l'envie*

*Modeste comme le génie  
Et simple comme la vertu*

*A Paris chez M. de la Roche, Libraire du Roi, Cour du Manège aux Thuilleries*

# THE MATHEMATICS TEACHER

Volume XXVII



Number 6

Edited by William David Reeve

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## The Old and the New Mathematics Education

By JOSEPH V. COLLINS

*State Teachers College, Stevens Point, Wisconsin*

MUCH HAS BEEN WRITTEN in recent years about McGuffey's Readers, but little or nothing of Ray's Series of arithmetics and algebras. In their day the Ray books had a large sale and were regarded as of as much importance as McGuffey's. The old arithmetic course differed from that of today in two exceedingly important respects. First, the old books employed a rather thoroughgoing inductive and deductive procedure in presenting topics. Second, nearly thrice as much time was devoted to arithmetic in the elementary course as now. The question is, which course is the wiser?

On the title page of the Ray books one found, "By the Inductive Method." This meant that when a new topic, like multiplication of fractions, was taken up, several examples were solved, easy ones first followed by more difficult ones. Next came a rule, generalized from the several solutions. Then came exercises to be solved by the rule. It may be noted that Ray was following the regular scientific procedure in learning, that is induction followed by deduction. The learner was expected to study the model solutions and see that the rule was a generalization from them.

Now when psychology became a fashionable study about thirty

years ago, professors of the subject began finding numerous faults with instruction in mathematics. Note that this was after there had been considerable curtailment in the time given to arithmetic in the elementary school. The professors said, the pupils skipped the model solutions, memorized the rules, and mechanically solved the problems by use of the rules. This may well be doubted save in exceptional cases, since it is far easier to follow examples than to follow generalized statements. But no matter, a hue and a cry were raised against rules, and authors promptly took them out of their books. In later texts the number of examples was cut down, dependence being placed on teachers' explanations.

Herbart's very suggestive steps in learning a topic are: *Preparation, Presentation, Comparison, Generalization, and Application*. Take as an example of following them, the teaching of the principle that multiplying or dividing both terms of a fraction by the same number does not alter its value. The preparation would be reviewing the terms, numerator and denominator, and the form of writing a fraction. Next, the presentation would consist in showing by use of concrete objects, as a pie diagram, a foot rule, that:  $1/2 = 3/6$ ;  $1/3 = 2/6$ ;  $2/3 = 4/6$ ;  $1/2 = 6/12$ ;  $1/3 = 4/12$ ;  $1/4 = 3/12$ ;  $1/6 = 2/12$ ;  $2/3 = 8/12$ ; etc. Now coming to the comparison (quite the most difficult step), the above equations are examined and it is found in every instance the terms on one side can be obtained from those on the other by multiplying or dividing both by the same number. The generalization consists in saying what is true in all the examples studied is always true. The book, or teacher, can say this is known to be true. Thus the presentation, comparison and generalization constitute the induction. Finally the generalization is changed into a rule and applied to the solution of exercises. This is deduction. Thus, if the general rule holds,  $9/12 = 3/4$ .

Now instead of all the above steps the new education passes directly from examples to exercises, thus cutting out comparison, generalization, formulating the rule, and applying the rule to exercises. Evidently the new procedure is far easier than the old, but there is little or no reasoning in it. Can we expect pupils themselves to *supply* this reasoning? Of course this procedure in the new education is followed in other subjects as well as in mathematics. If a student finds "in" means *not* in many words, will he generalize and later try to apply his generalization? The writer

finds pupils fail on a grand scale to learn the meaning of prefixes and suffixes and etymology in general, and the rules of grammar. It should be added that the charge just made does not apply to demonstrative geometry. When a figure is drawn for a proof, that figure stands for all such. Thus we have induction. In a proof when we refer back to a preceding proposition, we are using deduction. Thus both are used.

Now let us compare the old and the new mathematics from an entirely different standpoint. In the time of Ray three years out of eight of the elementary course were devoted to arithmetic. Nowadays  $1\frac{1}{4}$  years are supposed to be sufficient. At first blush one might think pupils would learn only  $12/5$  as much in three years as in  $1\frac{1}{4}$  years. But learning proceeds on a geometrical not an arithmetical progression basis. Thus pupils probably learn more than twice as much the second year as the first, and more than twice as much the third year as the second. Hence a three-year course may actually carry the pupil more than *seven* times as far as a one-year course. Well, who said three years out of the eight of the elementary school should be devoted to arithmetic? The American people. And who has said arithmetic should have only  $1\frac{1}{4}$  years? Answer, our educational theorists. How did this reduction in time come about? That is very easy to explain.

During the past fifty years it is said many more than 60 different branches have been added to the elementary and secondary curricula. It is easy to name some of the more important, as physiology and hygiene, civics, gymnastics, drawing, music, elementary science, manual arts, home economics, agriculture, etc., etc. Each of these can be sectioned, some into numerous studies. Now since neither the school year nor the day has lengthened, time had to be found for the new highly important branches. Mathematics was hit hardest in having time stolen from it. And what was the result?

Now mathematics including arithmetic is about the only subject in the curriculum that teaches abstract thinking. It was soon found that pupils with only  $1\frac{1}{4}$  years of arithmetic had great difficulty in learning ninth grade algebra and tenth grade geometry. The consequence was that these latter subjects had to be made easier as also the advanced work in arithmetic. Complaints to the superintendents and principals concerning algebra and geometry have become so numerous and forceful that there is a strong movement on to make these subjects elective. Unfortu-

nately two wrongs (cutting out arithmetic first, and later algebra and geometry) do not make one right.

Society is bound to pay the price for all this. The great body of our people nowadays can not reason quantitatively nor make required calculations accurately so as to guide wisely many of life's most important acts, and as a consequence pay severe penalties. It is too bad when men guess and guess wrong in essentials. Next all are poorly prepared for the study of science, pure and applied, and for advanced mathematics. As proof of the lowering of standards it can be said there are college algebras now on the market which are inferior to good secondary texts of thirty years ago. Does that indicate vaunted progress? One can not be even an intelligent citizen who can not check quantitatively on politician's claims.

What is the remedy? The late Chas. W. Eliot of Harvard saw the evils of our situation years ago, and proposed that we cut down the number of subjects offered and shorten many of those kept so as to secure sufficient time to teach thoroughly fundamental subjects, such as mathematics, science, and language.

Our teachers of secondary mathematics have been fighting bravely with their backs to the wall for a long time. They have done everything possible in the way of improvements in methods, inventing expedients, modernizing problem material, and so on. What they are really doing is trying to make bricks without straw and have had a more difficult task than the Israelites of old. Worst of all they seem to be fighting a losing battle in a most worthy cause. There is a strong movement on to make algebra and geometry elective subjects. Surely there is something wrong somewhere which remedied will solve the whole problem aright.

But on top of everything heretofore described the large increase in the number of high school students has lowered the average standard of attainment. This condition can probably be best improved by culling out with the customary means those pupils whose inferior ability or preparation would drag down any class they were in. These culls should be put in other classes and kept there until prepared.

High school teachers of mathematics in America, it seems to be up to you to show the country what is the matter and what the remedy is. If all join in the fight and work unitedly, you should win your fight.

# Development of Mathematics in Secondary Schools of the United States

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## PART IV

### MATHEMATICS IN THE JUNIOR-SENIOR HIGH SCHOOL PERIOD\*

#### *I. The Junior High School*

PRIOR TO THE latter part of the nineteenth century the energies of those interested in public education had been primarily directed toward the completion of the educational ladder. Elementary, secondary, and higher education had been placed under public control and to a large extent was financed by public taxation. In 1893 the Committee of Ten reported to the National Educational Association in favor of enriching the course of study in grades below the high school through the introduction of various subjects such as algebra, geometry, foreign languages, and natural sciences but their recommendations made no provisions for adapting these subjects to the abilities and needs of the children of the lower grades. During the early stages this movement for reorganization centered around the approximate equal division of time devoted to elementary and secondary education. The idea of dividing the six-year secondary school into junior and senior departments did not become a prominent one until the latter part of the first decade of the twentieth century.

The junior-senior high school, which experienced a rather rapid development and appears to have attained a permanent place in the organization of secondary education, entailed, in addition to administrative changes, a thorough readjustment of subject matter and methods of instruction. Investigations have

\* This is the fourth and last part of a discussion on the "Development of Mathematics in Secondary Schools of the United States." The third part appeared in the May issue of *THE MATHEMATICS TEACHER*.

shown that the reorganization of instruction did not keep pace with the changes in administration. The mathematical curriculum of the junior high school at the beginning of the third decade of the twentieth century was in a very decided state of flux and confusion, all of which was largely due to the absence of any generally accepted guiding principles and the consequent confusion in objectives.

The National Committee on Reorganization of Mathematics in Secondary Education, although aware of the need for a detailed syllabus which would give the best order of topics with the special time allotment for each, proposed, in their report of 1923, only a general outline by topics and stated that further experimentation was necessary before determining a standardized syllabus. This report stated that the junior high school pupil should be given a broad outline of the various fields of mathematics in order to give him an opportunity to find himself, to test his abilities and aptitudes, and to secure information and experience which would help him to wisely choose his later courses and ultimately his life work. With this idea in mind the mathematics of grades seven, eight, and nine, stated by topics rather than years, was to include the following:

*Arithmetic:* (a) the fundamental operations; (b) tables of weights and measures, including the metric system; (c) simple fractions; (d) facility and accuracy in the four fundamental operations; (e) short cuts in multiplication; (f) percentage; (g) line, bar, and circle graph; (h) arithmetic of the home; (i) statistics.

*Intuitive geometry:* (a) direct measurement of distances and angles, and approximate character of measurement; (b) areas, surfaces, and solids most frequently found in life, with the construction of corresponding formulas; (c) practice in numerical computation; (d) drawing to scale and use of square ruled paper; (e) geometry of appreciation; (f) simple constructions with ruler and compasses and T-square; (g) familiarity with simple forms of triangles and knowledge concerning the sum of the angles of a triangle, pythagorean relation, and simple cases of geometric loci in plane and space; and (h) informal introduction to idea of similarity.

*Algebra:* (a) formula, construction and use, and as the dependence of one variable upon another; (b) graphs and graphic representations in general; (c) positive and negative numbers; (d) equations in two unknowns and including applications in ratio and proportion; (e) algebraic technique; (f) factoring, including three simple types; (g) fractions; (h) exponents and radicals; and (i) stress of need for checking equations.

*Numerical trigonometry:* (a) definition of sine, cosine, and tangent; (b) their elementary properties as functions; (c) their use in solving problems; and (d) use of tables of those functions.

*Demonstrative geometry:* demonstrations of a limited number of propositions, with no attempt to limit the number of fundamental assumptions.

Although the committee refused to commit itself upon the arrangement of topics it did suggest five plans for distribution of the above material, hoping that they might prove helpful in the organization of mathematics curricula for the junior high school. The committee further recommended that the mathematics proposed for the grades of the junior high school be required of all pupils.<sup>1</sup> This program was based on the following fundamental assumption:

It is assumed that at the end of the sixth school year the pupil will be able to perform with accuracy and with a fair degree of speed the fundamental operations with integers and with common and decimal fractions.<sup>2</sup>

In 1930 Bridges tested 921 pupils of grades seven, eight, and nine in thirteen different schools situated in five different southern states. His report shows that with integers 85% of the group could successfully handle the addition problems, 88% subtraction, 61% multiplication, and 70% division; with decimal fractions 63% could carry out the addition, 68% subtraction, 43% multiplication, and 23% division; with common fractions 52% were able to solve the addition problems, 50% subtraction, 63% multiplication, and 43% division.<sup>3</sup> These results are quite in agreement with what Schorling found under somewhat similar circumstances.<sup>4</sup> Both of these studies seem to indicate that there is a need for revision of the fundamental assumption quoted above. At least they indicate that there is a striking need for the reteaching of the fundamental processes in the junior high school. In making an analysis of the errors made Bridges further states that, in addition to those errors which were unquestionably due to carelessness and guesswork, "a most frequent source of error was lack of knowledge of the fundamental combinations. Other errors of frequent occurrence were the omission or incorrect placing of the decimal point in multiplication and division. Carrying and especially borrowing proved difficult and zero combina-

<sup>1</sup> "The Reorganization of Mathematics in Secondary Education," The Mathematical Association of America, (1923) pp. 19-31.

<sup>2</sup> *Ibid.*, p. 20.

<sup>3</sup> W. A. Bridges, "Mathematical Ability of Pupils Entering the Junior High School," Unpublished Master of Arts Thesis, George Peabody College for Teachers, (1931), p. 11. The test used was the Compass Survey Tests in Arithmetic, Advanced Examination Form A.

<sup>4</sup> R. Schorling, "The Need for Being Definite with Respect to Achievement Standards," THE MATHEMATICS TEACHER, Vol. XXIV, (1931), p. 311.

tions were troublesome. In the work with fractions the pupils often confused multiplication and division with addition and subtraction."<sup>5</sup>

In 1927, four years after the publication of the report on reorganization, an examination of twenty-seven textbooks in junior high mathematics revealed a marked parallelism to the topics suggested, yet the whole course appeared as a mass of unrelated material with the recognition of the general acceptability of less than fifty per cent of the subject matter used by all of the authors.<sup>6</sup> Arithmetic and algebra ranked as most important when judged by amount of space devoted to the several topics, but geometry was found in the largest number of texts. Arithmetic was given first place in all of the seventh grade books except one, first in all of the eighth grade group except three, and was included in two ninth grade texts. Algebra appeared in the majority of the seventh grade texts, ranked second in seven of the eighth grade, and first in all of the ninth grade. Some phase of intuitive geometry was incorporated in all twenty-seven texts. Trigonometry was treated as such in three of the eighth grade group and six of the ninth grade.<sup>7</sup> A variety of topics were introduced probably as a result of the educational movement to provide sufficient usable mathematics for the student who did not expect to attend the senior high school. Practically all authors agreed that certain topics should be taught but there seemed to be no agreement as to the year or the semester in which they should be taught. There was rather general agreement upon the following: (a) formula with the simple equation as a tool in the solution of problems; (b) the graph of statistical data; (c) the angle; (d) the metric system; (e) elements of intuitive and constructional geometry; and (f) the study of the arithmetic of percentage and its elementary applications.<sup>8</sup>

A subsequent analysis of twenty-four texts and fourteen representative courses of study in junior high school mathematics revealed a consensus of opinion regarding some of the basic ideas

<sup>5</sup> W. A. Bridges, *op. cit.*, p. 15.

<sup>6</sup> Josephine Stone, "A Quantitative Analysis of Junior High School Mathematics Texts," Unpublished Master of Arts Thesis, George Peabody College for Teachers, (1927), p. 10.

<sup>7</sup> *Ibid.*, p. 53.

<sup>8</sup> *Ibid.*, p. 62.

around which the course of study should be organized. One year of arithmetic, at least one half year of intuitive geometry, one year of algebra, an introduction to trigonometry, and enough demonstrative geometry to convey the significance of demonstration were considered the essentials of a highly desirable course. The arithmetic should be that of life, house, shop, thrift, and everyday business; the intuitive geometry should be inventional and constructional; the emphasis in algebra should be on the formula, graph, and equation, and these concepts should be introduced in the seventh grade and continued throughout the three years.<sup>9</sup>

The arrangement of subject matter constituted a problem of at least as much difficulty as did its selection. The early tendency was simply to follow the compartment system of organization of the traditional high school course. This scheme has given way to a more popular plan which has continued to gain favor among educators in general. The new idea of arrangement of material whereby the courses of study were to be more closely related to each other was designated as "correlated," "composite," "unified," "coöperative," or more commonly as "general" mathematics. General mathematics for the junior high school has been defined as an "introductory, basic, exploratory course in which simple and significant principles of arithmetic, algebra, intuitive geometry, statistics and numerical trigonometry are taught so as to emphasize their natural and numerous interrelations."<sup>10</sup> This plan for the organization of mathematics can largely be traced to the effort of the Committee of Ten and the Committee on College Entrance Requirements during the latter part of the nineteenth century. Although there had been a few scattered instances of such treatment of mathematical subject matter prior to this time, the reports of these two committees seem to carry the first evidence of any concentrated thought devoted to its consideration. An address before the National Educational Association, in 1902, expressed the hope that a time would come when the secondary school course would comprise six years and when mathematics would not be limited by artificial boundaries as was

<sup>9</sup> Ruth Moncreif, "A Tentative Course of Study in Junior High School Mathematics," Unpublished Master of Arts Thesis, George Peabody College for Teachers, (1932), pp. 85-86.

<sup>10</sup> Raleigh Schorling, "General Mathematics," *THE MATHEMATICS TEACHER*, Vol. 20, (1927), p. 65.

the case in the study of algebra, geometry, and trigonometry.<sup>11</sup> Contemporaneous with this address the influence of Klein in Germany, Perry and Nunn in England, and Moore in America made the conditions more favorable for the rapid growth of general mathematics. In the Mathematics Section of the Central Association of Science and Mathematics Teachers so much interest was shown in a report on the unification of mathematics that a special investigating committee was appointed in 1907 and reappointed in 1908.<sup>12</sup> At the same time, however, a Committee of the Middle States and Maryland was preparing a course in algebra according to the traditional pattern.<sup>13</sup> In fact very little interest in this movement was shown by any of the Eastern Associations until several years later (1919). The National Committee of Fifteen on the formulation of a Geometry Syllabus, while not directly advocating correlated mathematics, did encourage the algebraic treatment of many geometric theorems.<sup>14</sup> Contemporaneous with and even prior to this period of increased interest in general mathematics, efforts were being directed toward making practical applications to classroom procedure of the principles advocated.<sup>15</sup>

Numerous experiments have been made to determine the status of this new organization of subject matter. McCormick has summarized the conclusions derived from the most significant of these:

- 1) There is no very clear or definite agreement among mathematicians and general educators as to what constitutes general mathematics.

- 2) General mathematics is gradually replacing the traditional type in the seventh, eighth, and ninth grades.

<sup>11</sup> Charles W. Newell, "Correlation of Mathematical Studies in Secondary Schools," *Proceedings of the National Educational Association*, (1902), pp. 488-492.

<sup>12</sup> "Report of the Meeting of the Mathematics Section of the Central Association of Science and Mathematics Teachers," *School Science and Mathematics*, Vol. 9, (1909), p. 92.

<sup>13</sup> "Report of the Committee on the Elementary and Intermediate Teaching of Mathematics in the Middle States and Maryland," *School Science and Mathematics*, Vol. 9, (1909), p. 900.

<sup>14</sup> "Report of the National Committee of Fifteen on a Geometry Syllabus," *School Science and Mathematics*, Vol. 11, (1911), p. 442.

<sup>15</sup> For a more detailed discussion of the above outlined development of general mathematics see: Clarence McCormick, *Teaching of General Mathematics in Secondary Schools*, Contribution to Education #386, Bureau of Publications, Teachers College, Columbia University, New York, (1929).

3) General mathematics provides training for college mathematics that is as good as, and perhaps better than, that of mathematics of the traditional type.

4) The indications are that general mathematics creates more interest in the subject than does traditional mathematics.

5) The opinions of high school teachers reveal a large number of reasons for teaching each type of mathematics. General mathematics is favored by many because of the wide variety of information given, because of the interest created, and because of its practical value. Traditional mathematics is favored by many others because of the more thorough knowledge imparted, because of its better organization, and because of the belief that it gives a better preparation for college.

6) Most of those persons who have specialized in methods of teaching mathematics are in favor of a more general type than has been offered.

7) Attempts are being made to correlate the different branches of mathematics. Textbook writers of today, however, are careful not to try to fuse material when an unnatural correlation results.<sup>16</sup>

As a result of the general mathematics movement in the junior high school we find that the mathematics courses have been changed from the formal arrangement to studies with concrete beginnings, less scientific rigor, more developmental and explanatory material, more practical exercises, better psychological development, more provisions for individual differences, and with the union and merging of related topics. This new type of organization of subject matter serves as an instrument for motivation, articulation and exploration. It helps to motivate the work through the many applications of one subject in the development of another and through the applications to problems from life situations. In the fusion of arithmetic with the more elementary phases of algebra and geometry not only is the transition from the arithmetic of the elementary school to the mathematics of the senior high school expedited, but excellent opportunity is afforded for the exploration of the student's aptitudes for the more advanced work. This effort to present the mathematics of the junior high school in a "fused" course has not been without its rather severe critics. There are those that feel that it is too much of a hodgepodge of superficialities that tends to general weakening of the subject content. Pupils in such classes have made the statement that the transfer from one topic to another is too rapid. Some critics have felt that too much emphasis is given to continuity of subject matter and not enough attention paid to the significance of the individuality of different topics.

<sup>16</sup> *Ibid.*, p. 162.

One suggestion to offset such difficulties has been to follow the pattern set by the European schools and present the entire content of secondary mathematics in a parallel course, thus preserving the integrity of the subject matter and supplying in the method of presentation the desired continuity. Such a scheme would call for a teacher so well trained that she would not be confined to textbook organization and who would be thoroughly familiar with opportunities and principles of coördination as well as with practical illustrations of the utilitarian and cultural values of the subject.

A recent plan suggested to strengthen the pulse of junior high school mathematics is as follows:

## FOR SUPERIOR STUDENTS

## FOR INFERIOR STUDENTS

*Grade 7*

Geometry (a full half year) and arithmetic

Geometry (a full half year) and arithmetic

*Grade 8*

Algebra (a full half year) and arithmetic

Formula, graph, equation; arithmetic; geometry including trigonometry of the right triangle.

*Grade 9*

Algebra, advanced arithmetic (computation), and numerical trigonometry—for the upper 40 per cent of students in this grade.

Junior business training with incidental arithmetic, taught by the commercial department. (This ends the prescribed course in mathematics for the lower 60 per cent.) A half-year non-mathematical course in generalization and logical argumentation designed to preserve so far as possible the outcomes of algebra and geometry.<sup>17</sup>

While the junior high school period should provide a coördinated training in the fundamentals necessary for the minimum essentials of social efficiency, it must furthermore function as a period of apprenticeship for the more advanced work of the senior high school. It must consequently supply that substance and coherence necessary for proper orientation and progress in the later years of more specialized endeavor.

<sup>17</sup> Ralph Beatley, "Coherence and Diversity in Secondary Mathematics," Eighth Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, (1933), p. 213.

## II. *The Senior High School*

Just as the organization of the junior high school is based upon the four fundamental principles: (1) articulation; (2) exploration, revelation, and guidance; (3) interpretation of environment; and (4) motivation:<sup>18</sup> the primary function of the senior high school is to provide for specialization and the pursuance of one's aptitudes and interests. Such a difference in the basic philosophy of the two institutions demanded a fundamentally different program of curriculum organization. The National Committee on Mathematical Requirements recognized this difference in the recommendations presented in their report of 1923. They formulated a body of general aims supplemented by two lists of specific aims differentiated for the two levels of instruction. These general aims were recommended as of functional value to all secondary mathematical instruction and were grouped as follows:

### I. Practical Aims.

- 1) Undisputed utility of the fundamental processes of arithmetic in the life of every individual.
- 2) Understanding of and ability to use intelligently the language of algebra.
- 3) Development of the ability to use and understand algebraic methods.
- 4) Ability to understand and interpret correctly graphic representation.
- 5) Familiarity with more common geometric forms; development of space perception and exercise of spatial relations.

### II. Disciplinary Values.

- 1) Ideas or concepts in terms of which quantitative thinking of the world is done.
- 2) Development of the ability to think clearly in terms of those concepts.
- 3) Acquisition of correct mental habits and attitudes.

### III. Cultural Aims.

- 1) Appreciation of beauty in geometrical forms of nature, art, and industry.
- 2) Logical reasoning and discrimination between the true and false.
- 3) Appreciation of the "power of thought, the magic of the mind."<sup>19</sup>

The Committee recommended that the specific aims for the mathematics of the senior high school be provided for through a body of elective material which should be open to all pupils

<sup>18</sup> J. M. Glass, "Tested and Accepted Philosophy of the Junior High School Movement," *The Junior-Senior High School Clearing House*, vol. 7, March (1933), pp. 329-339.

<sup>19</sup> "The Reorganization of Mathematics in Secondary Education," *The Mathematical Association of America*, (1923), pp. 6-10.

who had satisfactorily completed the required work of the junior high school. Fully realizing that the method of organization of this material could be a bit more elastic in nature than that for grades seven, eight, and nine, and also thoroughly cognizant of the fact that no one best plan had been determined the Committee suggested four different plans, any one of which might be used for the purpose of more efficiently organizing the instructional content of the mathematics of grades ten, eleven and twelve.

The principle purposes of the instruction in plane demonstrative geometry were "(1) to exercise further the spatial imagination of the student, (2) to make him familiar with the great basal propositions and their applications, (3) to develop understanding and appreciation of a deductive proof and the ability to use this method of reasoning where it is applicable, and (4) to form habits of precise and succinct statement, of the logical organization of ideas and of logical memory." To realize these purposes the usual list of topics were suggested, with the exception of the omission of the formal theory of limits and the idea of motion in the demonstration of theorems. Expression was given to the belief that, by organizing and selecting the great basal theorems, a reduction of from thirty to forty per cent could be made in the number of formal proofs, thus providing opportunity for the treatment of an increased number of well-graded and carefully selected exercises.

The proposed course in algebra was to include: (1) solution of problems in linear and quadratic equations including formulae from science and common life; (2) different methods for solving quadratic equations with a brief discussion of complex numbers, also the graphical solution of equations of higher degree than the second; (3) the algebraic solution of linear equations in two or three unknowns and the graphical solution of linear equations in two unknowns; (4) the graphic and algebraic solution of a linear and a quadratic equation and of two quadratics that contain no first degree term and no  $xy$  term; (5) exponents, radicals, and logarithms; (6) arithmetic and geometric progressions; and (7) the binomial theorem with proof for positive integral exponents and extension by statement to negative and fractional exponents.

It was recommended that the emphasis in solid geometry be placed upon the development of the spatial imagination of the student that he might acquire a knowledge of the fundamental

space relationships and the power to work with them. The following program was submitted as one for accomplishing the desired results; (1) propositions related to lines and planes, and to dihedral and trihedral angles; (2) mensuration of solids, with numerous computations using established formulae; (3) spherical geometry; and (4) similar solids. It was further suggested that desirable simplification and generalization could be introduced into the treatment of mensuration theorems by employing the Cavalieri and Simpson theorems and the Prismatoid Formula, without necessarily rigorously demonstrating them.

Trigonometry in addition to logarithms, solution of right and oblique triangles, radian measure, graphs of trigonometric functions, the derivation of the fundamental relations between the functions and their use in proving identities and in solving easy trigonometric equations, was to include the use of the transit in surveying and the sextant in astronomical observations incident to the measurement of local time. A course in elementary calculus was suggested for the twelfth grade which was to include: (1) a general notion of the derivative as a limit; (2) application of derivatives to easy problems in rates and maxima and minima; (3) simple cases of inverse problems; (4) approximate methods of summation leading to integration; and (5) applications to simple cases of motion, area, volume, and pressure. It was recommended that this work be largely graphic and that the necessary technique be limited to the treatment of algebraic polynomials.

Additional electives such as elementary statistics, mathematics of investment, shop mathematics, surveying and navigation, and descriptive or projective geometry were suggested for schools where there was a need for such work and where the conditions warranted their inclusion in the curriculum. It was also recommended that extensive use be made of historical and biographical material in the entire teaching program to lend interest and significance to the subject matter studied.<sup>20</sup>

Having noted a wide divergence in certification requirements for teachers of secondary mathematics in the United States and fully cognizant of the impossibility of setting up any generally acceptable standard for the preparation of teachers, the Committee proposed a standard of certification which they considered highly desirable. In brief outline it was as follows:

<sup>20</sup> *Ibid.*, pp. 34-39.

- 1) Graduation from a standard four-year college or its equivalent.
- 2) Credit for at least the following courses in mathematics:
  - a) Plane and spherical trigonometry.
  - b) Plane analytic geometry and the elements of analytic geometry of three dimensions.
  - c) College algebra.
  - d) Differential and integral calculus with applications;
  - e) Synthetic projective geometry.
  - f) Scientific training in geometry.
  - g) Scientific training in algebra.
- 3) At least theoretical and practical physics and chemistry.
- 4) Four semesters of theoretical professional courses.
- 5) Satisfactory performance of the duties of a teacher of mathematics in a secondary school for a period of not less than twenty semester hours.<sup>21</sup>

Another great influence in the reform of the teaching of mathematics in the secondary schools is that of the College Entrance Examination Board which has been a bit radical in many of its reforms. A Commission appointed by the Board made an outstanding and influential report in 1923 in which it recommended the elimination from the list of college entrance requirements of the extended and useless manipulation of polynomials, the limiting of problems in factoring to three types, the simplification of the requirements in fractions, increased recognition of the use of the formula and graph, simplification of the work in surds, and the introduction of numerical trigonometry. In the plane geometry syllabus only eighty-nine theorems were included and of these approximately one-third were all that were to be required for examination purposes. The Commission further recommended that plane and solid geometry be combined into one course and that more work of a practical value be offered in mensuration.<sup>22</sup>

The change in objectives in the teaching of algebra and the change in the emphasis on topics made possible a change in the nature and sequence of subject matter. Much of the obsolete material has been eliminated and new material introduced for the purpose of enriching and improving the course. The first instance of such change was the introduction of the graph which began to receive attention during the first decade of the present century. A still more recent innovation, and by far the most important

<sup>21</sup> *Ibid.*, pp. 507-508.

<sup>22</sup> D. E. Smith, "A General Survey of the Progress of Mathematics in our High Schools in the Last Twenty-five Years," *First Yearbook of the National Council of Teachers of Mathematics*, (1926), pp. 12-14.

of changes in algebraic instruction, is the introduction of the concept of functional dependence. Although, since the beginning of the twentieth century, the constant proposal of all reform suggestions has been to replace the formal symbolism of algebra by the notion of functional relationships, it remains true that textbook writers and teachers of algebra fail to utilize the vast possibilities of this unifying element of constructive mathematical thought.

The many changes in the textbooks in plane geometry reflect the influences of the criticisms brought against them. While there is by no means a complete agreement as to the organization of subject matter, the authors, who seem to have felt the general influence of the reform movement, have worked toward the same ultimate goal, that of leading the pupil to independent discovery. The criticisms that have been directed toward instruction in geometry may be briefly summarized and classified as follows: (1) failure to establish a concrete basis for demonstrative geometry; (2) lack of opportunity for immediate application of the principles learned; (3) failure to construct texts based on psychological principles; (4) failure to emphasize training in logical thinking; (5) failure to correlate geometry with arithmetic, algebra, trigonometry, and sciences; (6) failure to recognize the existence of individual differences.

An examination of texts revealed the following evolution of the subject matter in plane geometry:

- 1) Efforts on the part of recent writers to establish a concrete basis for the beginning of demonstrative geometry may be summarized as follows:
  - a) longer and more informal introductions;
  - b) the introduction of developmental exercises;
  - c) concrete illustrations of geometric principles;
  - d) practical applications;
  - e) an increase in pictures illustrating the use of geometry;
  - f) the use of drawing instruments.
- 2) The decrease in the number of propositions with "ready-made" proofs, the increasingly large number of exercises of all types, and the large numbers of exercises placed for immediate application indicate that textbook writers are beginning to realize the importance of applying the geometric principles learned.
- 3) From the point of view of psychology of learning it is apparent that significant advances have been made. The contents of the older texts consisted mainly of definitions, propositions, and corollaries. No appeal was made to the pupils' instinct of discovery. Leading pupils into independent discovery through experimental work; giving challenge to his challenge, ingenuity, and imagination; assisting him to an understanding of the principles of geometry through practical applications; all of these constitute sufficient evidence that writers of the more

recent texts are giving careful consideration to the psychological structure of geometry as well as the logical.

- 4) The recognition of training in logical thinking as a most important result of the study of plane geometry is evident in the changes that have been made in the treatment of the proposition, such as the incomplete proof, the unproved theorem and unsolved problem, the analytic treatment, and in the hints for solution and plan of attack on theorems; in the grouping of theorems; in the significant increase in the number of exercises; in the grading of exercises according to difficulty; and in the question type of exercise.
- 5) Attempts to correlate plane geometry with other branches of mathematics are seen in the increased number of algebraic and trigonometric exercises as well as exercises taken from fields other than mathematics.
- 6) The problem of individual differences is partly taken care of by the inclusion of optional material, the differentiation of propositions and exercises, and by following the outlines for maximum and minimum courses as presented by the authors.<sup>23</sup>

The effort to evolve a democratic program of mathematical instruction adapted to the education of the masses which resulted in the "general mathematics" type of organization of subject matter for the junior high school, produced significant changes in senior high school instruction. The content of the mathematics for these later years of secondary instruction is more individualistic in nature than that of the three previous years and, as a consequence, has not been so readily adapted to a fused method of organization. One of the most difficult problems that confronts the teacher of senior high school mathematics is that of so organizing and presenting the content to preserve its intrinsic individuality yet introduce the desired continuity. One such program based upon a differentiated program for superior and inferior students in the junior high school recommends no mathematics in grades ten, eleven, and twelve for inferior students and suggests the following parallel program of instruction for the upper fifty per cent of the pupils of these grades: grade ten, demonstrative geometry, and after this has been well started a continuation of algebra (begun in the ninth grade) in parallel with it; grade eleven, demonstrative geometry and algebra in parallel; grade twelve, a comprehensive course in trigonometry, solid geometry, algebra, analytic geometry, and some reference to the methods of the

<sup>23</sup> Dell Terry, "An Analysis of Some Plane Geometry Textbooks," Unpublished Master of Arts Thesis, George Peabody College for Teachers, (1932), pp. 32-34.

Ellen M. Freeman, "Textbook Trends in Plane Geometry", *School Review*, Vol. XL, (1932), pp. 292-293.

calculus. The entire course is to be completed with a very comprehensive examination.<sup>24</sup>

As the educational program has evolved to keep pace with an ever expanding democratic progressive social order, mathematics has always been able to adjust its offerings to conform to the current philosophy of curriculum revision. As a subject it has lost some of its prestige as an integral portion of the instructional machinery of secondary education, largely due to the failure of teachers and administrators to formulate its content into topics that are vital, timely, and pertinent to the age and stage of culture in which the social order is moving. The future of mathematics in the secondary school curriculum lies in the ability of the teachers of mathematics to realize the full value of the subject and its close contact with the structure of civilized progress, and in their ability to so organize and present its content that the student may thoroughly appreciate its historical significance, present potentiality, and future challenge.

<sup>24</sup> Ralph Beatley, *loc. cit.*

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#### SALE ON YEARBOOKS

The National Council Yearbooks—two to nine inclusive—may now be obtained postpaid for \$11.00 from *The Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York, N.Y.* See first page of this issue for particulars.

## The Value of Analytics and Calculus in the Secondary School

By HOWARD F. FEHR

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"TO BE MORE SPECIFIC, I am opposed to the introduction into the secondary school of the calculus in any form, of projective geometry, of analytic geometry as such, or of any special stress on the concept of function." Thus Ralph Dennison Beetle<sup>1</sup> of Dartmouth College rejects in part the recommendations of the National Committee, which say that "The one great idea which is best adapted to unify the course is that of functional relation" and which also include calculus in every plan of arrangement of material for the twelfth year. Having taught analytics and calculus as such, for the past four years to twelfth year students and having stressed the function concept in the eleventh and twelfth years, I am in direct opposition to Professor Beetle's opinion. In this article are presented some of the advantages that have resulted from this teaching which seem to make it desirable as a required part of the advanced mathematics in the secondary school.

The type of student in the twelfth year mathematics class might well be studied first. These pupils in most high schools have had a year of elementary algebra including numerical trigonometry, a second year of plane geometry and a third year of intermediate algebra including logarithms and the solution of triangles (in many cases oblique triangles are included). While for the most part the first year is compulsory for all pupils, the second and third years are elected either because the pupil needs these courses for college entrance or because he has the talent and liking for the subject. The fourth year is elected by those pupils who have a decided preference for Mathematics, who have succeeded well in it during the past three years and who will continue to study or use it in their post-high-school life. These pupils are the most gifted of the mathematics students in the high school and as a class are superior to most college freshman classes in mathematics.

<sup>1</sup> "Advanced Mathematics in the Secondary Schools," *THE MATHEMATICS TEACHER*, February, 1934; page 89.

That this is so is evident from the fact that most colleges now require for entrance but two or two and one-half years of mathematics. After completing these requirements, many pupils drop their study of mathematics for two or one and one-half years before entering college. The freshman class in college, then, consists of very many pupils who have forgotten a great deal that must be relearned for successful future work and who are not particularly interested in mathematics for its own value. In South Side High School, Newark, N. J., the average I. Q. of students taking fourth year mathematics during the past three years has been 122, the range being 103 to 145. This course is completely elective. With such a group far more can be done *and should be done* than with the average class in freshman mathematics where even now in many colleges a course in analysis is given including functions, analytics, and the calculus.

As to how much algebra, geometry, and trigonometry we should teach in the high school, no definite answer can at present be given. Of course all three years of the senior high school might be spent on algebra or geometry alone without exhausting either subject. It is my contention that the more abstract and theoretical phases of these subjects belong to the senior and graduate work in college and are far more appreciated there, after the student has had a general knowledge of the various fields of mathematics. It is only then that one can truly appreciate treatises such as Chrystal's Algebra or Hobson's Trigonometry. To take special parts of these subjects while the pupil is studying the elementary phase, leaves the subject disconnected and leads to no definite unity of concept. If a student has mastered the elementary plane geometry, no direct gain can be accomplished at that time by introducing modern geometry of the same type. It is of greater cultural value that the student learn of other geometries and how they operate. In short, no rigorous logical development of algebra or geometry is possible in the high school or early college period and is not desirable until the pupil has a more comprehensive view of mathematics and has mastered the mechanics. To learn to manipulate these mechanical processes is easy, but to learn the fundamental concepts necessary for their rigorous development is a process involving time.

If we accept the statement that the pupil should obtain an adequate understanding of the underlying theory as well as the

mechanics of mathematics, we are led to the function concept with compelling force. Without it the study of the quadratic in algebra becomes a mere mechanism of substitution. The same is true in trigonometry in which, if we do not stress the dependence of the ratios upon the size of the angle and the use of sin, cos, etc., in place of  $f(x)$  as indicating particular dependences, we have nothing but a mere set of rules and operations. The solution of triangles is so often merely this, that our tendency today is to stress the analysis of these *functions* and complete the study of the solution of triangles in as short a time as possible. That is why the radian, the graphs, and the line values of these functions are introduced at the beginning of the study of the subject.

*The concepts of function and limit are not grasped in a day, a week, a month, or a year but are attained only after a continuous growth through several years of study.* This is one reason why the inter-dependence of quantities should be shown to mathematics students as early in their school career as possible, and through continuous application the students of average and higher ability will develop this concept, fundamental to all mathematics. Professor Kasner of Columbia University, speaking recently at a conference at Teachers College, advised that the idea of limits be introduced to pupils at the age of from ten to twelve years and gave examples of problems to be used. If students first hear of limits in their second year of college upon taking up the study of calculus, they do as most students in the past have done, learn the tricks of differentiating and integrating without knowing what it is they are doing. In fact the pupil is so busy learning and applying the new symbolism and formulae, that he has no time left to study the underlying theory of limits and theorems of Rolle, the Mean, Taylor, and others.

In the past we have dodged the teaching of the theory of limits whenever it arose in geometry or trigonometry, putting the theorems on a postulational basis. The idea behind this is that limits form too difficult a subject for high school pupils. This is only an assumption, as no real attempts have been made (at least not in high school textbooks) to give an adequate introduction. If the theory is presented at first intuitively, beginning with infinite geometric progressions, followed by examples from elementary mechanics on velocity and acceleration and finally by simple algebraic functions, one will find the students grasping an idea

which enables them to appreciate the physical relations expressed in formulas, to understand the power of a derivative applied to rates and maxima and minima, to grasp the behavior of infinitesimals (differentials), and to use with great satisfaction the integral calculus in finding areas and volumes of plane and solid figures. It is of value to note that only the simplest algebraic functions enter into the integration of all the areas and volumes treated in ordinary geometry.

When we teach our classes, we cannot tell who are going to continue their mathematical education in college, who will not go to college, or if they do enter college what particular institution it will be. In public high school very few of the students are definite on this matter even towards the end of their last year. We must therefore give them the best mathematical education they are capable of achieving, regardless of whether they will enter a higher institution or not, and regardless of the type of instruction they will receive in the higher institution. In every case this education, to my mind, must be climaxed in the secondary school, as it is in Germany, England, and France, with analytics and the calculus, and welded together with the function concept. For those who continue in college the great gain will be in a growing concept, fertilized by the expansion of the analytics, the calculus, and their fundamental theorems, and reaching a climax in the beautifully rigorous theory of functions.

No real teacher would attempt to teach the calculus in the high school to gain the pupil's respect or to parade his knowledge. In fact the problem is to find high school teachers who are equipped to teach the analytics and the calculus. Those who are equipped, having seen the vastness of the field of mathematics, readily recognize the elementary character of the part they would teach. They teach as men and women who believe mathematics is a great liberalizing and disciplinary subject and who desire that all those capable shall have an opportunity to share in the secrets of this glorious knowledge. Many a pupil has been won as a student of mathematics because a scholarly inspiring teacher has led him away from the mechanics of algebra and the rote of geometry into the understanding and appreciation of the power of the elementary calculus.

Even as intuitive geometry prepares and makes easier the way

for demonstrative plane geometry, just so an elementary introduction to the analytics and the calculus paves the way to a more rigorous treatment of these subjects. When a student has discovered how simple it is to locate a locus in the analytics compared to the tedious evaluation of the intermediate algebra; when he easily discovers the locus of points equidistant from a given line and point, which he could never accomplish in Euclidean geometry; when he learns how to build a rectangular solid of given volume with the least amount of material, which is to him an almost impossible task in algebra; and when he has found the area between curves, which the ordinary geometry would never solve, he has received a knowledge and insight that can never be equalled in the secondary school by a deeper study of algebra, geometry, or trigonometry. And more, such a student steps into college with a distinct advantage over those who have had the formal solid geometry, trigonometry, and advanced algebra, because of the familiarity with the symbolism and material they are to meet. If they do not go to college, they have had an experience which will permit them to do further reading and study in mathematical and technical fields, which would have been forfeited by limiting the study to the traditional subjects.

In mathematics a student who has pursued algebra, geometry, and trigonometry for three years as separate units, has the right to see where they all unite in application. There is no better review or application of the algebraic operations involving factoring, fractions, radicals and exponents, and equations, than in finding derivatives, simplifying them, and using them in problems involving rates, maxima, minima. These topics alone convince the student that the intensive drill on these algebraic processes was well worth the while and the effort. In the theorems on limits a student recognizes the value of the training in logical thinking which he received in the study of plane geometry. The facts that he uses from plane geometry are so few that their value in the study of geometry is minimized. On the other hand a comparison of locus in Euclidean geometry with that of analytical geometry, and the application of the latter in the calculus leaves no doubt in the pupil's mind of the superiority of coordinate geometry.

To demonstrate the particular values that accrue from the study of the function concept, analytics, and the calculus in the high

school it would be necessary to tell in more detail what material is used and especially *how it is presented*. This I should like to do in some future article, but here it seemed best to present a few of the general values of such study. I have appended a concise syllabus of the twelfth year mathematics in the order in which it has been presented in our school during the last two years. The values from this study may be summarized as:

1. A student who has acquired the concept of function can more readily, vividly, and intelligently apply it to the study of physical phenomena.

2. A student recognizes in the calculus a tool indispensable in modern engineering and science, and is the more appreciative of the modern scientific achievements which he enjoys.

3. A student familiar with the notation and simpler operations of coordinate geometry and calculus who does not continue his education in higher institutions can far more readily pursue the subject without classroom instruction, while the student who does go on to college adapts himself more readily to the new methods of presentation.

4. A student familiar with the function concept has a general method of attacking geometric problems instead of the "Bag of tricks" which he used in Euclidean geometry.

5. As a child matures so should his ideas of algebra, geometry, and trigonometry; and this growth is best obtained for him by showing their application in the higher branches of mathematics rather than in advanced parts of the same field.

6. A student having had such a course can read books and technical papers on recent scientific developments with better understanding. Finally and very important,

7. These simpler methods of the coordinate geometry and the calculus unify the mathematical knowledge of the student since they make indiscriminate use of practically all the mathematics the student has previously learned.

Here follows an outline of the work as I have presented it to my own classes:

TWELFTH YEAR MATHEMATICS  
SOUTH SIDE HIGH SCHOOL, NEWARK  
*B-Semester*

1. Permutations and combinations. (No elements repeated.)
2. Solid geometry, book VI, lines and planes in space.

3. Functions; the notation, evaluation, kind. Algebraic and trigonometric functions; graphs of functions; Radians.
4. Elementary trigonometric relations, reduction of angles, review of logarithms, solution of triangles.
5. Vectors, relative velocities, complex numbers, DeMoivre's theorem,  $n$ -roots of any number.
6. Trigonometric analysis—sum, difference, multiple, partial angles; sums and differences of functions; Identities, equations, inverse functions; general values.

#### *A-Semester*

1. Elementary analytics—straight line, circle, parabola, ellipse, hyperbola; their definition, application and derivation of equations.
2. Limit—progressions,  $\pi$ ,  $\epsilon$ ,  $\sin x/x$ ; theorems, rational functions.
3. Extension of DeMoivre's theorem, expansion of sine and cosine into series, compound interest, law of organic growth, hyperbolic functions.
4. Derivatives, rate problems, maxima and minima; differentials.
5. Integration as the antiderivative; as a limiting sum.
6. Applications of integration—area, arc, surface and volume of revolution.
7. Solid geometry completed with the use of the integral calculus; the prismoid formula and Euler's formula for polyhedrons.
8. Polynomials—elementary theory of equations—solution of cubics.
9. Elementary probability and statistics.

The following issues of the *Mathematics Teacher* are still available and may be had from the office of the *Mathematics Teacher*, 525 West 120th Street, New York.

- Vol. 14 (1921) Jan., Feb., April, May.
- Vol. 16 (1923) Feb., May, Dec.
- Vol. 17 (1924) April, May, Dec.
- Vol. 18 (1925) April, May, Nov.
- Vol. 19 (1926) May.
- Vol. 20 (1927) Feb., April, May, Dec.
- Vol. 21 (1928) Mar., April, May, Nov., Dec.
- Vol. 22 (1929) Jan., Feb., Mar., April, May, Nov., Dec.
- Vol. 23 (1930) Jan., Feb., Mar., April, May, Nov., Dec.
- Vol. 24 (1931) Feb., Mar., April, May, Oct., Dec.
- Vol. 25 (1932) Jan., Feb., Mar., April, May, Oct., Nov., Dec.
- Vol. 26 (1933) Feb., Mar., April, May, Oct., Dec.
- Vol. 27 (1934) Jan., Feb., Mar., April, May, Oct.

**Price: 25c each.**

## Evaluating "The Mathematics Teacher"

By CONSTANCE McCULLOUGH

*College of Education, University of Minnesota*

THE PROFESSIONAL JOURNAL in the special subject matter fields has varied functions to perform in the service of education. It must endeavor to strike a balance between the professional and the scholarly interests of its subscribers. For the teacher it presents materials for classroom use and methods of teaching with a view to improving instructional procedures. By discussing the aims of education and the contribution of the particular subject to the fulfillment of them, it provides a guiding philosophy for the direction of the teacher's efforts, and a basis for curriculum construction and revision for those charged with supervisory or administrative responsibility. By reporting the findings of research, it provides both teacher and supervisor means for the constant alteration of materials and methods, leading to progress in the entire field. By including reports of activities in related professional organizations, it keeps the reader informed concerning the problems confronting others in his field, and makes him aware of the shifting centers of professional emphasis at the moment. From the point of view of scholarship in his academic field, the magazine enriches the experience of the reader by providing historical backgrounds, by giving increasing depth to his knowledge, and by affording him broader vision of the scope of his subject.

The value of the professional journal to its reader depends upon the fulfillment of these various functions. A good index of the effectiveness of such a journal is the relative emphasis which it gives to each of them in its monthly offering. Another element indicative of its influence is the character of its contributing personnel. To what extent, for instance, do teachers in service avail themselves of the opportunity afforded by such a magazine to exchange viewpoints and compare experiences? To what extent do educational experts find it a useful means of sharing the results of their research and the outcomes of professional study and discussion? To what extent, finally, does the scholar in the field of

\* THE MATHEMATICS TEACHER is your journal. Please send your comments on this article to the editor for possible publication in the TEACHER and to guide him in the choice of materials.—Editor.

pure mathematics contribute through its pages to the breadth of scholarship in the field? For the purpose of an evaluation based upon these criteria, eleven issues of *THE MATHEMATICS TEACHER*, beginning January, 1933, were chosen for analysis. Eighty-five articles were studied and classified according to the author's affiliation with high school or college, the mathematical or professional nature of the article, the type of subject, and the apparent aim of the writer. The results are as follows:

Twenty-two (25.88%) contributors to *THE MATHEMATICS TEACHER* over the period January, 1933, through March, 1934, were secondary school teachers. Of the remaining sixty-three, all except eleven who were not identified with a particular school were affiliated with normal schools, colleges, or universities.

The accompanying table shows that contents were directed largely toward the interests of the classroom teacher. A substantial percentage (36.47%) of the articles dealt with classroom methods. While certain topics pertained to administrative problems, these constituted only one-fourth of the total number of articles. Twenty-four (28.24%) articles were classified as mathematical rather than professional in content, and were concerned chiefly with the history of mathematics. Although there was no particular uniformity in the offerings of various issues of *THE MATHEMATICS TEACHER*, each presented a biographical sketch of the mathematician represented on the frontispiece.

It will be noted that seven (8.24%) of the eighty-five articles fell under the category of reports of scientifically controlled investigations. As a matter of curiosity, the *1930-31 Bibliography of Research Studies in Education*, the last of these bulletins to be published by the government, was perused for studies in mathematics. Of the sixty-nine studies in algebra and geometry listed for that year, fifty-nine seemed of immediate interest to the teacher in the field. Twenty-six contained pertinent findings on the efficacy of certain teaching methods. In the monograph entitled *Selected References in Education, 1933*, published at the University of Chicago, Breslich listed fourteen published studies in mathematics. Of these three had appeared in *THE MATHEMATICS TEACHER*. If the number of studies in the year 1930-31 may be considered a rough approximation of that produced in 1933, relatively few significant research studies undertaken in secondary mathematics reach the teacher in the field.

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Since a determination of the writer's aim is necessarily arbitrary, no attempt will be made here to present quantitative data respect-

*Number and Percentage of Articles on Certain Types of Subject*

Types of subject	Number and percentage of articles							
	Authorship						Total	
	Normal School or College		High School		Misc.			
	No.	%	No.	%	No.	%	No.	%
Professional.....	37	43.53	16	18.82	8	9.41	61	71.76
Methods.....	18	21.18	13	15.29	2	2.35	33	38.82
Classroom.....	17	19.99	13	15.29	1	1.18	31	36.47
Aims.....	1	1.18	2	2.35			3	3.53
Content.....	16	18.82	11	12.91			28	32.94
Scientific investigations	2	2.35	1	1.18			3	3.53
Research.....	1	1.18			1	1.18	2	2.35
Curriculum.....	13	15.29	2	2.35	2	2.35	17	19.99
Aims.....	1	1.18					1	1.18
Content.....	12	14.12	2	2.35	2	2.35	16	18.82
Scientific investigations.....	3	3.53	1	1.18			4	4.71
Personnel.....	3	3.53					3	3.53
Reports.....	3	3.53	1	1.18	4	4.71	8	9.42
National Council.....	2	2.35			3	3.53	5	5.88
Other professional organizations...	1	1.18	1	1.18	1	1.18	3	3.53
Mathematical.....	15	17.65	6	7.06	3	3.53	24	28.24
History.....	13	15.29	1	1.18			14	16.47
General.....	2	2.35	1	1.18			3	3.53
Biographies of mathematicians.....	11	12.91					11	12.91
Solutions of Problems.....	2	2.35	2	2.35	1	1.18	5	5.88
Mathematical Plays and Skits.....			3	3.53	2	2.35	5	5.88
TOTAL.....	52	61.18	22	25.88	11	12.91	85	100.00

ing the aims observed. In the order of their relative frequency of occurrence, however, the apparent aims were guidance of teacher attitude and opinion, presentation of methods and ma-

terials for practical use, and promotion of teaching as a profession through encouraging research and urging the maintenance of standards in teacher preparation.

An analysis such as this raises a number of questions. To what extent, for instance, should a professional magazine concern itself with descriptive accounts of the experiences of the secondary school teacher? To what extent should it give voice to the judgment of those whose positions offer a broader perspective and longer vision? At this particular time should more attention be given to problems of course reconstruction and the different patterns in use in various school systems? Should the findings of research, upon which the general betterment of teaching efficiency depends, be reported for ready adoption and verification in classrooms throughout the country? How much scholarly background can be taken for granted in the average reader of such a periodical? To what extent should a professional journal be devoted to the enrichment and extension of that knowledge? This report is presented in the hope that it may stimulate some formulation and expression of opinion as to what readers would like to find in *THE MATHEMATICS TEACHER*.

## PLAYS

Back numbers of *The Mathematics Teacher* containing the following plays may be had from the office of *The Mathematics Teacher*, 525 West 120th Street, New York.

A Near Tragedy. Miller, Florence Brooks, XXII, Dec. 1929.

An Idea That Paid. Miller, Florence Brooks, XXV, Dec. 1932.

If. Snyder, Ruth L. XXII, Dec. 1929.

Mathematical Nightmare. Skerrett, Josephine, XXII, Nov. 1929.

Mathesis. Brownell, Ella, XX, Dec. 1927.

The Eternal Triangle. Raftery, Gerald, XXVI, Feb. 1933.

The Mathematics Club Meets. Pitcher, Wilimina Everett, XXIV, April 1931.

More Than One Mystery. Russell, Celia A., XXVI, Dec. 1933.

Price: 25¢ each.

# The Changing Content of Ninth Grade Mathematic Texts

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By RUTH OLSON

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NINTH GRADE MATHEMATICS has for some time offered a problem to curriculum makers. Many have held that a subject which occasioned such a large percentage of failures was ill-fitted to hold its place in our courses of study, and numerous efforts have been made to adapt it to the needs of the child. Texts have been written which aim to integrate algebra with arithmetic and intuitive geometry. Problems based directly upon the experience of the child have been included. Topics such as highest common factor and cube root have almost disappeared, and in their places one may find bits of trigonometry and statistics in graphic form. To furnish a picture of the changes which have occurred in the make up of ninth grade mathematics in the last few decades and to show to what extent these changes agree with modern theories of education was the purpose of an investigation which is reported here.

## SOURCES OF DATA

In order to obtain materials for comparison, textbooks were selected as the best source of information. It is true that textbooks are influenced by the requirements for college entrance, but it is equally true that they represent the material which the student covers during the course. Methods may vary, school systems may add or omit topics in formulating their plans, teachers may emphasize or treat hastily certain portions of the year's work; but in spite of these minor variations, the text still furnishes the fundamental basis for the work covered in the course.

Textbooks from three different periods were studied. The first period (1895-1900) is a time when secondary education was for the few rather than for the many, and when its function was still largely college preparatory. The second period (1910-1915) is characterized by the changes accompanying the rapidly increasing high school population and the changing ideals of secondary education. It marks the time when the movement for reform in teaching of mathematics was gathering the impetus which re-

sulted in the organization of the National Committee on Mathematical Requirements in the fall of 1916. The recent period is represented by the years from 1924 to 1929 during which current practices in the teaching of ninth grade mathematics became established. The particular books chosen for study were selected upon the basis of the frequency with which the text was used. The number of schools adopting the text rather than total number of books sold was used in determining frequency, and in this way weight was given to the factor of geographical distribution, and the effect of city adoptions minimized.

#### TOPICS UPON WHICH EMPHASIS HAS BEEN CONSTANT

The large number of topics occurring in ninth grade mathematics necessitates some method of grouping, and a consideration of the recommendations of various authorities<sup>1</sup> makes it evident that the material may be grouped in four main divisions. These divisions are shown in Tables I and II. Table I contains the topics which the authorities cited either explicitly or implicitly assume to be present in the course. It is evident that the topics included in this table were found in most of the texts examined, and the exceptions are easily explained. The recent text which did not include evaluation as a separate type gave that type of work in the study of the formula; and the three recent texts which did not include powers and roots gave a full allotment of space to radicals and surds. Since the last mentioned topic contains many ideas in common with powers and roots, there is no actual omission in spite of the evidence. The omission of reviews in five texts of the middle period and one of the recent period is also more apparent than actual. It means that in the texts where the omission occurred, reviews of specific topics are found—

<sup>1</sup> *The Reorganization of Mathematics in Secondary Education*, Mathematical Association of America, Inc., National Committee on Mathematical Requirements, 1923, pp. 19-31.

Requirements of College Entrance Board, National Council of Teachers of Mathematics, First Yearbook, Teachers College, Columbia University, 1926, pp. 9-14.

D. E. Smith and W. D. Reeve, "Objectives in the Teaching of Junior High School Mathematics," National Council of Teachers of Mathematics, Second Yearbook, Teachers College, Columbia University, 1927, pp. 173-227.

Raleigh Schorling, "A Course of Study Based upon Objective Studies," National Council of Teachers of Mathematics, First Yearbook, Teachers College, Columbia University, 1926, pp. 78-80.

that is, a review of factoring or review of long division—and as such are included under the topic for which they furnish supplementary material.

When the average percentage of space allotted to each of the various topics is considered, there is a fairly constant amount for the three periods. In a few cases, the averages run high for the early period, but this is to be expected because the books of the early period, for the most part, were analyzed only to the point where quadratic equations were introduced. Such a procedure

TABLE I  
*Topics Receiving Relatively Constant Emphasis*

Topics	Per cent of space			No. of books		
	Early	Middle	Recent	Early	Middle	Recent
Nature of algebra. . . . .	7.04%	4.25%	5.80%	10	10	10
Fundamental operations with integers. . . . .	14.90	11.46	11.30	10	10	10
Fundamental operations with fractions. . . . .	10.75	6.42	6.89	10	10	10
Simple parentheses. . . . .	1.38	1.28	1.54	10	10	10
Evaluation. . . . .	0.73	0.82	0.92	10	10	9
Simple factoring. . . . .	3.60	3.63	3.55	10	10	10
Special products. . . . .	2.62	2.58	2.46	10	10	10
Linear equations. . . . .	13.02	12.05	12.82	10	10	10
Simultaneous equations	5.38	5.40	4.80	8	10	10
Powers and roots. . . . .	2.76	0.96	0.61	10	10	7
Arithmetical square root	1.84	1.88	1.81	9	10	10
Reviews. . . . .	2.31	3.87	7.08	10	5	9
Totals. . . . .	66.33	54.60	59.31			

necessarily gives a greater proportion of space to the topics which were antecedent to quadratics in their books. The totals found in Table I indicate no consistent tendency toward decrease or increase in the amount of material used. There is a slight decrease between the early and middle periods, probably due to the inclusion of a wider range of topics during the middle period; and this is followed by a somewhat smaller increase between the middle and recent periods. These differences, however, are relatively small and point toward the conclusion that the emphasis placed upon topics in this group has been constant.

#### TOPICS UPON WHICH EMPHASIS HAS CHANGED

Table II contains the topics upon which the emphasis has not been constant, and for convenience in analysis they are divided

TABLE II  
Topics Receiving Varying Emphasis

Topics	Per cent of space			No. of books		
	Early	Middle	Recent	Early	Middle	Recent
A. Decreasing Emphasis						
Quadratic equations...	1.88%	5.71%	4.98%	2	8	9
Literal equations.....	1.94	2.06	1.67	8	9	10
Radicals and surds.....	5.57	5.36	4.80	8	10	10
Ratio and proportion...	1.12	2.49	2.08	3	9	10
Variation.....		0.53	0.72		4	6
Highest common factor and lowest common multiple.....	4.55	1.87	0.68	10	10	9
Theory of exponents.....	1.63	2.33	0.76	7	10	8
Totals.....	16.69	20.35	15.69			
B. To be Eliminated						
Nested parentheses....	0.20%	0.25%	0.13%	6	7	3
Complex factoring.....	2.73	2.70	0.80	9	10	8
Complex fractions.....	0.82	0.69	0.42	8	10	8
Complex simultaneous equations.....	2.71	3.53	1.49	5	10	6
Square root of polyno- mials.....	1.68	1.13	0.77	10	10	9
Cube and higher roots...	3.59	0.35	0.02	9	4	1
Various theorems.....	2.37	1.68	0.63	8	5	1
Nature of roots—dis- criminant.....	0.59	0.14	0.13	2	2	2
Imaginary and complex numbers.....	0.81	0.56	0.13	5	8	3
Permutations and com- binations.....		0.17			1	
Series and progressions...	1.12	0.89	0.04	2	2	1
Totals.....	16.62	12.09	4.56			
C. Increasing Emphasis						
Formula.....	0.33%	3.10%	4.41%	2	10	10
Illustrations.....		1.98	2.81		6	7
Historical and biograph- ical notes.....		0.91	0.90		3	5
Graphs.....		4.25	2.82		10	9
Trigonometry.....		0.27	3.21		1	9
Logarithms.....		0.66	0.47		1	1
Tables (trigonometric)...		0.07	1.23		1	8
Significant figures.....		0.01	0.10		1	2
Statistical measures....		1.58	3.97		9	10
Intuitive geometry.....		0.18	0.51		2	6
Slide rule.....						
Totals.....	0.33	13.01	20.43			

into three groups: A, B, and C. Group A consists of those topics which modern educators would have treated with less emphasis than formerly. The percentage of space allotted to these topics shows that practice has not coincided entirely with theory. Radicals and surds give evidence of a small but steady decrease in the amount of space allotted to them, and there is a marked decrease in the case of highest common factor and lowest common multiple; but with few exceptions the tendency has been toward an increase followed by decrease. This condition is probably due to the fact that the number of older texts mentioning these topics is comparatively small. However, a consideration of the number of books including topics in this group shows that there has been an actual increase in most cases. Although there is basis for saying that these topics are receiving decreasing emphasis, especially in the recent period, it is true, nevertheless, that their inclusion is becoming more general. This group may then be considered as an established part of the course, secondary in importance to the group previously discussed.

Group B consists of material which modern theory would exclude from ninth grade mathematics. The first six topics are simple enough in theory, but they give rise to difficult problems. Besides, the effort put forth by a ninth grade pupil in order to solve such problems far overbalances the benefits derived. The remaining topics are difficult in theory—beyond the understanding of the average ninth grade pupil; and, with the first six, belong in advanced algebra rather than in a beginning course. It was not their difficulty, however, which placed them in this group. It was the fact that they did not further any of the aims of mathematics teaching in the ninth grade.

The general tendency as shown by the totals is toward the elimination of these topics, although some are dropping out more rapidly than others. Cube and higher roots have almost disappeared; series and progressions edged their way in during the middle period and then dropped out entirely; complex factoring and the consideration of various theorems (remainder, inequalities) have decreased to one-fourth of the space originally allotted to them. In only three instances (nested parentheses, permutations and combinations, and the more complex simultaneous equations) is there an increase followed by a decrease. In the first two of these topics these variations are negligible; in the case of the more com-

plex simultaneous equations, the increase in the middle period may be explained by the fact that in the early period quadratic equations were not included, and consequently simultaneous equations involving quadratics were also missing.

If consideration is given to the topics in the group which were omitted by more than two-thirds of the books, there were three omissions in the early period, three in the middle period, and seven in the most recent. The newer books seem to show more restraint in selecting material. The only topics included by a majority of the texts in each of the three periods are complex factoring, complex fractions, and the square roots of polynomials. These all show consistent decreases, although that of complex fractions is relatively smaller than that of the other two. The fact that the College Entrance Board still specifies simpler types of complex fractions in its requirements probably explains this situation. Although no study was made of the difficulty of the material, it was evident that there was decreasing difficulty as well as decreasing space allotment.

The totals from Group C also indicate that the advice of educational authorities is being given practical expression. Both in the percentage of space and in the number of books there has been a general increase as the years went by. Only one topic in this group, the formula, is found in the books of the early period; and the small amount of space given to it puts it on the same plane with the other topics which were non-existent in ninth grade mathematics at that time. There are only two instances of decrease. The first is in the presentation of graphing and this is accompanied by a comparatively large increase in the amount of material given on elementary statistics. This shows a shift of emphasis from the formal type of graphing to the practical rather than any decreasing interest in the topic as a whole. The other topic showing this tendency is logarithms and it may be disregarded because of the fact that only one book in each period includes this topic.

The greatest increases were found in the amount of space allotted to trigonometry and its accompanying aid—tables. Both in the amount of space and the number of books, there have been decided gains. A similar increase might have been expected in the case of intuitive geometry, and there are three possible reasons for its non-appearance. First, the list of the ten most widely used

books for the recent period did not include a single text in general mathematics. Secondly, while the possibility of teaching intuitive geometry is always present for any teacher who wishes to use it, the burden of the work is laid upon her, as very little actual material of this type is included in the texts. In the third place, the selection of material for inclusion under this topic was made in such a way that geometric illustrations, without explanations concerning their geometric nature, and statements of geometric facts, unaccompanied by illustrative material were excluded. Of the remaining topics, logarithms, slide rule, and the discussion of significant figures rank low both in amount of space and in the number of books including them, while illustrations and notes of a biographical and historical nature seem fairly well established.

In considering this group as a whole, there can be no question that the tendency is toward its inclusion. Some of the topics have been more widely accepted than others; but it may be assumed that the entire group, representing as it does the effort to increase the immediate and practical values of the subject, has found a permanent place in the ninth grade course of study.

#### THE MODERN TEXTBOOK IN NINTH GRADE MATHEMATICS

The general aspects of the content of ninth grade mathematics as shown by the comparisons just made, together with the consideration of the recommendations of various authorities, indicate that a desirable textbook for use at the present time would be one in which the contents were distributed in the following manner:

The topics in Table I would occupy about 60% of the space. For the topics in A of Table II, the space allotment would probably range from 14 to 17%. The problems in ratio and proportion would be of a practical nature; quadratics would be simple in type, and probably more space would be given to solution by means of the formula than was customary in the older texts. The treatment of the theory of exponents, of highest common factor, and of lowest common multiple would be such that they would be included only as explanatory materials for the topics to which they are related.

Of the topics in B of Table II, the only one which could not be eliminated is complex fractions; and the only urgent reason for including it would be the necessity for conforming to college entrance

requirements. The other omissions would release 4 to 5% of the total number of pages for other topics, probably those of C, Table II. This last group would then occupy between one-fifth and one-fourth of the total space. There would be a reasonable amount of material in the form of historical and biographical notes and portraits which serve to connect the subject of mathematics with other fields; and the formula, statistical measures and graphing, trigonometry, and intuitive geometry would be treated in a way that would bring home their practical value.

This textbook would be the best that could be assembled in the light of the collected data, but it would represent only the present. New theories are constantly being formulated. Never before has there been more unrest in the field of ninth grade mathematics than at the present time, and such unrest of necessity brings change. But mathematics has during the past thirty-five years responded to changing ideas in a way hardly to be expected of a subject weighted with inertia of long accepted traditions; and what it has done in the past, it can continue to do in the future.

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## Jean Baptiste le Rond d'Alembert

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*Born in Paris, November 16, 1717*

*Died at Paris, October 29, 1783*

ON NOVEMBER 16, 1717, a gendarme found a baby who had been abandoned near the church of Saint Jean le Rond in Paris. The boy was given the name of the church and was boarded out by the parish in the home of a glazier who lived in the vicinity. At a later time, his father made provision for his education, and when he became famous, it is said that his mother tried to get in touch with him, but he refused her overtures and stayed with his foster mother although she was not in entire sympathy with his career. Ball quotes her as saying, "You will be nothing but a philosopher. And what is a philosopher? He is a fool who torments himself during his life so that people will talk about him when he is dead."

On taking his bachelors degree in 1735, Jean le Rond assumed the name of d'Alembert. He studied law, medicine, and mathematics, doing most of his work in the latter subject in the interval from 1743 to 1754. The remainder of his life was largely devoted to the French encyclopedia which was initiated by Diderot and d'Alembert and to which d'Alembert contributed the introduction as well as articles on philosophy and mathematics.

In 1740, d'Alembert was made a member of the French Academy and after 1754, he served as its permanent secretary. He wrote on mathematical astronomy, the calculus and its applications, differential equations, and dynamics. He dedicated one of his works to Frederick the Great and in return he was offered a pension and an invitation to come to Berlin. D'Alembert spent only a short time there and later he refused the invitation of Catherine II to come to Russia to supervise the education of her son.

Cajori quotes an incident that is significant of d'Alembert's character. He had given a severe criticism of a mediocre piece of work and he defended his action by saying "I'd rather be rude than bored."

VERA SANFORD

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## NEW BOOKS

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*Geometry Professionalized for Teachers.*

By Halbert Carl Christofferson, Oxford, Ohio, 1933. Published by the author. 203 pp. Price \$1.50 net to teachers.

It is one of the encouraging signs of the time that such a book as this should be written and that its publication should be assured. Geometry, like other branches of mathematics, is under critical inspection, often on the part of those who are manifestly incapable of passing judgment as to the merits of the subject. On the other hand teachers possessed of scholarship, not only in the field of education, but also in that of mathematics, occasionally are found to pass calm judgment on the real questions at issue—What is the purpose in teaching geometry, and how is that purpose best achieved? To find a general educator who can discover flaws in a subject like geometry is an easy matter—simply go to any school of education and find a man to whom the subject was poorly taught. To find a mathematician who can discover flaws in a subject like the theory of education is equally simple—simply look in any class, for example in the theory of functions of a complex variable, and find a man whose teaching is that of most college instructors. Neither one is possessed of a judicial frame of mind that qualifies him for fair criticism.

Dr. Christofferson represents neither of these extremes. He knows mathematics and he is familiar with the best that the general educators have to offer. He has no sympathy with the methods of teaching geometry that were used in the nineteenth century; neither has

he any brief for the general educator who would destroy a subject like geometry without offering something equally valuable to replace it.

Furthermore, Dr. Christofferson is a man who has read widely both in the field of modern education and in that of modern geometry. If he shows any definite bias it is on the educational side and, unfortunately, on the side of American authors to the exclusion of the more conservative, if not more scholarly, European writers. Nevertheless it is to his credit that he has impartially considered the opinions of the best of our American contributors to the subject in hand along with those whose opinions do not, in general, exercise much influence.

The major problem which Dr. Christofferson sets for himself concerns the necessary training of teachers of geometry. For the solution of this problem he sets forth four objectives:

- (A) The mathematical objectives;
- (B) The professional objectives;
- (C) The professional assumptions upon which the course is built;
- (D) The specific mathematical and professional objectives.

It is not feasible in a brief review to consider these in detail, but it is possible to mention a few features of special importance to teachers. Stated succinctly these seem to this reviewer to include the following:

1. The postulates of geometry may all be false and some of them certainly are so, from the standpoint of higher mathematics.

2. All that geometry can claim by way of absolute truth is that it proves that

if the assumed postulates are true, then the proofs based upon them are true.

3. Scientifically, it is desirable to limit the number of postulates as far as possible. Practically, considering the fact that geometry is being taught to boys and girls in their teens, it is better to increase the number of postulates. For the same reason it is necessary to state them in a simpler form than would be the case if we were teaching advanced students who might be capable of understanding the irreducible minimum. In particular, Dr. Christofferson follows the trend in many schools by postulating the congruence theorems. There is no objection to this, although the difficulty is met by the single postulate of moving figures in space, and teachers should feel free to postulate these theorems if they so desire. Bertrand Russell's statement that superposition is "a tissue of nonsense" is itself nonsense if the necessary postulates precede this method of proof.

Some of the author's assertions relating to postulates, however, might well be rewritten. For example, "No mathematician would think of postulating angle trisection" is true or false according to the meaning, and this is obscure. Any mathematician would feel free to postulate that there must be a point which bisects a line segment; that a line must exist which bisects an angle; that lines exist which trisect an angle; and that a regular heptagon exists. This is quite different from postulating the construction of these figures. The matter is discussed by the author on page 40, but not as clearly as might be desired, for there is a marked difference between "hypothetical construction" and "hypothetical existence," as in the case of a regular heptagon. In order to ask the size of an angle in such a figure as this regular polygon manifestly need not await the construction of the figure.

4. The discussion of the number of propositions in a workable list in plane and solid geometry is very helpful. It embodies the recent results of an important subject. It is to be hoped that it will silence those who speak about "covering plane geometry," as if the subject could or should ever be "covered."

It is quite impossible, in the space allowed to the reviewer, to speak of the merits of the book as a whole. Suffice it to say that these merits far surpass the minor points of criticism which have been adduced. The latter are mentioned not as matters of adverse comment on this book, but of school usage at the present time. The book will be very helpful to teachers of geometry, and it offers to them a bibliography which will be of great assistance in forming a high-school library.

One of the easiest and generally the most useless of efforts made by the reviewer of a book is that of finding fault over inconsequential blemishes. Such blemishes can be found in this and most other books. Just why some footnotes, even in the same section, are marked by asterisks and others by numbers is a mystery, but a matter of no moment. That the old, and etymologically incorrect, spelling "parallelopiped" should have been retained is more unfortunate because it sets a wrong standard for teachers. It is also unfortunate that "compass" is used for "compasses." Like "O.K." and "It is me," there is usually no misunderstanding in such usages, but, after all, the schools may be expected to favor higher standards. In the same way it is to be regretted that so much sanction is given to expressions like " $a$  over  $b$ ." Of course there is authority for it, but is the phrase necessary, and are the sanctions the best? To speak of " $b$  under  $a$ " would seem as valid, and also as unnecessary.

The ancient expression " $a$  into  $x+y$ " for  $a(x+y)$  has much greater sanction but is equally unnecessary and is wisely omitted in this book.

The historical notes are not always as correct as might be desired. It is not to be expected that the Sumerian discoveries should have been mentioned on page 15, because they are too recent to have place. The "earliest records in Egypt in 2300 B.C." is too precise, as is the statement that the Rhind papyrus is "largely" a copy of an older document dating back to 2300 B.C. Many of the dates, too, should be stated as being mere approximations, as in cases like "Pythagoras (580-500 B.C.)." The reader should not, however, judge an excellent work by such trifling errors.

On the whole, the book is a real contribution to the educational-mathematical literature of the time and is worthy of the careful study of the teacher.

DAVID EUGENE SMITH

*Health Through the Ages.* By C.-E. A. Winslow and Grace T. Hallock. Published by the School Health Bureau, Welfare Division, Metropolitan Life Insurance Company.

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The basis for the free distribution of this chart is one copy to a classroom.

*Health Through the Ages* by C.-E. A. Winslow and Grace T. Hallock, 64 pp., ill., 1933, School Health Bureau, Welfare Division, Metropolitan Life Insurance Company, New York.

*Light and Shade*: A chart to accompany *Health Through the Ages*, 1934, School Health Bureau, Welfare Division, Metropolitan Life Insurance Company, New York.

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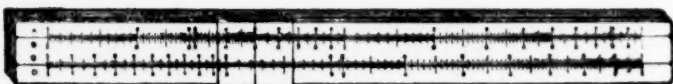
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